5 th
Polish
Combinatorial
Conference

ABSTRACTS

Będlewo, September 22–26, 2014

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LIST OF TALKS

Seyedeh Narjess Afzaly

Extremal graphs without given cycles

The extremal graph theory was introduced by Turán in studying the maximum number of edges in graphs without cliques of given orders. We consider extremal graphs not containing cycles of given lengths.

For $C \subseteq \{3, 4, 5, \ldots\}$, define $\mathrm{ex}(n, C)$ to be the maximum number of edges in a graph on n vertices without any cycle of length k for all $k \in C$. Denote by $\mathrm{Ex}(n, C)$ the set of all *extremal graphs* with respect to (n, C), i.e., the set of all graphs on n vertices and with $\mathrm{ex}(n, C)$ edges without any cycle of length k for all $k \in C$.

In search for $\mathrm{ex}(n, C)$ and $\mathrm{Ex}(n, C)$, we present an algorithm to produce all extremal graphs for $n \leq 64$ and $C \subseteq \{4, 6, 8, \ldots, 32\} \cup \{3, 5, 7, \ldots\}$. The algorithm can also produce extremal graphs with extra restrictions on maximum and minimum degree and the number of vertices of minimum degree.

Based on this algorithm we could improve the boundaries on n and C for which $\mathrm{ex}(n, C)$ and $\mathrm{Ex}(n, C)$ are known.

Mahdi Amani

Ranking and unranking algorithm for neuronal trees in B-order

In this paper, we present two new ranking and unranking algorithms for neuronal trees in B-order. These algorithms are based on a generation algorithm which is given for integer sequences corresponding to neuronal trees by Pallo (1990).

A neuronal tree is a rooted tree with n external nodes (leaves) whose internal nodes have at least two children. These trees are used in computational neuroscience for modeling the connections between neurons in brain, and are also called neuronal dendritic trees. Up to our knowledge no other ranking and unranking algorithms are given for integer sequences corresponding to neuronal trees in B-order. The time complexity of the presented ranking and unranking algorithms for neuronal trees with n leaves are $O(n)$ and $O(n \log n)$ respectively.

Sylwia Antoniuk

On a very sharp threshold for collapse of random triangular groups

The random triangular group $\Gamma(n, p)$ is the group given by a random presentation with n generators and a random set of relators R, each of length three, and such that we choose each relator to be present in R independently with probability p. We are interested in the behavior of the random group in the $\Gamma(n, p)$ model when $p = p(n)$ and the number of generators n goes to infinity. In particular, we show that the property of collapsing to the trivial group admits a very sharp threshold.

Joint work with Ehud Friedgut, Tomasz Łuczak and Jacek Świątkowski.

Małgorzata Bednarska-Bzdęga

Waiter–Client games

Let n, q be positive integers and A be a monotone graph property. In the Waiter–Client A-game (known in the literature as a Picker–Chooser game) in each round Waiter selects exactly $q+1$ free edges of the complete graph K_n and offers them to Client. Then Client selects one of them which he keeps and the remaining q elements are claimed by Waiter. The game ends when there is no free edge left. Waiter tries to force Client to build graph with property A. We will talk on some recent results on Waiter–Client games.

Joint work with Dan Hefetz, Michael Krivelevich and Tomasz Łuczak.

Csilla Bujtás

On the game domination number of graphs with given minimum degree

In the domination game, introduced by Brešar, Klavžar and Rall in 2010, Dominator and Staller alternately select a vertex of a graph G. A move is legal if the selected vertex v dominates at least one new vertex – that is, if we have a $u \in N[v]$ for which no vertex from $N[u]$ was chosen up to this point of the game. The game ends when no more legal moves can be made, and its length equals the number of vertices selected. The goal of Dominator is to minimize whilst that of Staller is to maximize the length of the game. The game domination number $\gamma_g(G)$ of G is the length of the domination game in which Dominator starts and both players play optimally.

In this talk we establish an upper bound on $\gamma_g(G)$ in terms of the minimum degree δ and the order n of G. Our main result states that for every $\delta \geq 4$,

$$\gamma_g(G) \leq \frac{30\delta^4 - 56\delta^3 - 258\delta^2 + 708\delta - 432}{90\delta^4 - 390\delta^3 + 348\delta^2 + 348\delta - 432}\, n.$$

Particularly, $\gamma_g(G) < 0.5139n$ holds for every graph of minimum degree 4, and $\gamma_g(G) < 0.4803n$ if the minimum degree is greater than 4. Additionally, we prove that $\gamma_g(G) < 0.5574n$ if $\delta = 3$.

Alexey Chuprunov

Practical approach in telecommunications for theory of distribution for distinguished random particles

This paper derives some asymptotic theorems of combinatorial distributions for distinguished random particles. It was shown that these theorems consequences' may be used as analytical performance boundaries for different multichannel telecommunication systems. Functional of packet error rate from signal-noise ratio was infered for special case of uniformity of different channel structures, received messages and environmental interferences.

Joint work with Dmitry Chickrin and Petr Kokunin.

Marek Cygan

Using FPT tools to approximate k-Set Packing

One of the most natural optimization problems is the k-Set Packing problem, where given a family of sets of size at most k one should select a maximum size subfamily of pairwise disjoint sets. A special case of 3-Set Packing is the well known 3-Dimensional Matching problem. Both problems belong to the Karp's list of 21 NP-complete problems. In this talk we show a $(k+1+\epsilon)/3$-approximation algorithm for k-Set Packing, resulting in a 1.34-approximation for 3-Dimensional Matching, which in turn improves over the $(1.5+\epsilon)$-approximation algorithm of Hurkens and Schrijver [SIDMA'89].

Interestingly, to obtain new local search based approximation algorithm we use tools from fixed parameter tractability, namely colour coding and graphs of bounded pathwidth.

Sebastian Czerwiński

Harmonious coloring of hypergraphs

Let $H = (V, \mathcal{E})$ be a hypergraph, and let $c : V \to \{1, 2, \ldots, r\}$ be a *strong coloring* of the vertices of H, which means that no two vertices of the same edge have the same color.

For every edge $X \in \mathcal{E}$, let $p(X)$ denote a *color pattern* of X, that is, the set of colors appearing on X. A coloring c is called *harmonious* if $p(X) \neq p(Y)$ for every pair of distinct edges $X, Y \in \mathcal{E}$.

We denote by $h(H)$ the least number of colors needed for a harmonious coloring of H, and call it occasionally the *harmonious number* of H. This notion arose as a natrual generalization of harmonious coloring of graphs introduced by Harary.

The case of multisets is a natural generalization of *legitimate* coloring of projective planes studied by Alon and Füredi.

There are other graph coloring problems related to this topic. Indeed, if H is a hypergraph dual to a simple graph G, then the notion of harmonious coloring of H corresponds to the vertex-distinguishing edge coloring of G, introduced by Burris and Schelp.Thus, $h(H) = \chi'_s(G)$, where $\chi'_s(G)$ is the related chromatic parameter.

The following two results are obtained using entropy compression argument.

Theorem 1 *Let G be a d-regular graph on n vertices. Assume that G has a perfect matching. Then there is a vertex-distinguishing edge coloring of G using at most $K\sqrt[d]{n}$ colors, where K is a constant depending only on d given by the following formula:*

$$K = (2^{2d-1}(d-1)!(d-1))^{\frac{1}{d}} + \left(\frac{2^{2d-1}(d-1)!}{(d-1)^{d-1}}\right)^{\frac{1}{d}}.$$

For cubic graphs the above theorem gives $6\sqrt[3]{2n}$ which is currently best bound.

Theorem 2 *Let k and Δ be fixed positive integers. Then there exists $m_0 = m_0(k, \Delta)$ such that every k-uniform hypergraph H of maximum degree Δ with m edges satisfies $h(H) \leq \frac{k}{k-1}\sqrt[k]{k \cdot k!\Delta m}$, provided $m \geq m_0$.*

Kinga Dąbrowska

Multicolour Ramsey numbers for cycles or paths versus some subgraphs of wheels

The k-colour Ramsey number $R(G_1, G_2, \ldots, G_k)$ is the smallest integer n such that in arbitrary k-colouring of edges of the complete graph K_n a subgraph G_i in the colour $i, 1 \leq i \leq k$, is contained.

Recently many results for Ramsey numbers of cycles versus fans and wheels have been obtained. For instance Burr and Erdős in [1] showed that $R(C_3, W_n) = 2n + 1$ for $n \geq 5$, Radziszowski and Xia in [2] applied a simple method for obtaining the Ramsey numbers $R(C_3, G)$, where G is either a path, a cycle or a wheel. Surahmat, Baskoro and Tomescu in [3] showed that $R(C_n, W_m) = 2n - 1$ for even m and $n \geq 5m/2 - 1$ and $R(C_n, W_m) = 3n - 2$ for odd m and $n > (5m - 9)/2$.

In this paper we present similar results for some subgraphs of wheels versus paths or cycles.

References

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Michał Dębski

The mystery of sequences of large radius

A *cyclic k-radius sequence* over an n-element alphabet A is a sequence in which every two elements of A occur within distance k of each other (where the distance is defined as the difference of indices of terms and is calculated cyclically – so that the distance between the first and last term is 1). For example, Euler cycle in a complete graph (viewed as a sequence of vertices) is a cyclic 1-radius sequence.

Our goal is to determine the minimum possible length of such sequences. We focus on cyclic cn-radius sequences over $[n]$, where c is a constant satisfying $0 < c < 1$ and n goes to infinity.

We construct sequences of length matching the trivial lower bound (which is $n\left\lceil \frac{1}{2c} \right\rceil$) for some values of c and improve this bound in some other cases.

There is still a huge gap between mentioned results. Unknown ground starts when $c \in \left(\frac{1}{4}, \frac{1}{3}\right)$ and appears in a similar form in infinitely many other intervals. For smaller c, the problem seems to be related to the famous Prime Power Conjecture.

Joint work with Zbigniew Lonc. Research supported by the Polish National Science Center, decision no. DEC-2012/05/B/ST1/00652.

Sergei Evdokimov

Coset closure of a circulant S-ring and schurity problem

Let G be a finite group. There is a natural Galois correspondence between the permutation groups containing G as a regular subgroup, and the Schur rings (S-rings) over G. The problem we deal with in the paper, is to characterize those S-rings that are closed under this correspondence, when the group G is cyclic (the schurity problem for circulant S-rings). It is proved that up to a natural reduction, the characteristic property of such an S-ring is to be a certain algebraic fusion of its coset closure introduced and studied in the paper. Basing on this characterization we show that the schurity problem is equivalent to the consistency of a modular linear system associated with a circulant S-ring under consideration. As a byproduct we show that a circulant S-ring is Galois closed if and only if so is its dual.

Jan Florek

On Barnette's Conjecture

Let $\mathcal{B}$ denote the class of all 3-connected cubic bipartite plane graphs. A conjecture of Barnette states that every graph in $\mathcal{B}$ has a Hamiltonian cycle.

A *cyclic sequence of big faces* is a cyclic sequence of three or more different faces, each bounded by at least 6 edges, such that two faces from the sequence are adjacent if and only if they are consecutive in the sequence. Suppose that F_1, F_2, F_3 is the unique proper 3-coloring of the faces of $G^* \in \mathcal{B}$. We prove that if every cyclic sequence of big faces in G^* has a face belonging to F_1 and a face belonging to F_2, then G^* has the following properties: H^{+-}: If any two edges are chosen on the same face, then there is a Hamiltonian cycle through one and avoiding the other, H^{--}: If any two edges are chosen which are an even distance apart on the same face, then there is a Hamiltonian cycle that avoids both.

Moreover, if every cyclic sequence of big faces in G^* has a face belonging to F_1, then G^* has a Hamiltonian cycle.

Alan Frieze

Variations on Maker–Breaker graph games

We present two variations on the Maker–Breaker paradigm. In Walker–Breaker one of the players is constrained to follow a walk. We prove results on the number of vertices visited by Walker. In the second variation there is a bound placed on the degree in the graph induced by edges selected by both players.

Maciej Gawron

Twins in words and permutations

Axenovich, et. al [1] considered the following problem. For a word w let $f(w)$ be the largest integer k such that there are two identical, disjoint subsequences (called *twins*) of length k in w. How fast the following function

$$T(n,\Sigma) = \min\{f(w)|w \in \Sigma^n\},$$

grows (where Σ is a finite alphabet)? As a main theorem they proved that $2T(n,\{0,1\}) = n - o(n)$. The proof is based on regularity lemma for words.

We consider analogous problem for permutations. Let $\sigma \in S_n$ be a permutation over the alphabet $\{1,2,\ldots,n\}$. Two disjoint subsequences $\{a_i\}_{i=1}^k$, $\{b_i\}_{i=1}^k$ of $(1,2,\ldots,n)$ are called twins if the following condition

$$\sigma(a_i) \leq \sigma(a_j) \iff \sigma(b_i) \leq \sigma(b_j)$$

holds for all $i,j = 1,2,\ldots,k$. Let $g(\sigma)$ be the largest integer k such that there are twins of length k in σ. We will give some bounds on the asymptotic of the following function

$$P(n) = \min\{g(\sigma)|\sigma \in S_n\}.$$

In order to give some bounds we will use probabilistic methods, Lovász local lemma and Erdős-Szekeres theorem. Exact asymptotic is left open.

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Przemysław Gordinowicz

Cops, robbers and ordinals

The game of cop and robber, introduced by Nowakowski and Winkler in 1983, is played on a graph G by two players, one controlling a cop and the other one a robber, both positioned on vertices of G. The players alternate moving their pieces to distance at most 1 each. The cop win if she capture the robber, the robber wins by escaping indefinitely.

There are a few characterisations of finite cop-win graphs but in the infinite case only one of them still works: for the finite graph G consider the sequence of relation $\{\leq_i\}_{i\in\mathbb{N}}$ on $V(G)$, given by the conditions

1. $u \leq_0 v$, when $u = v$,

2. $u \leq_i v$, if for all $x \in N[u]$ there exists $y \in N[v]$ such that $x \leq_j y$ for some $j < i$.

where $N[v]$ denotes (closed) neighbourhood of vertex v.

This sequence is increasing, thus there exists a minimum integer k such that $\leq_k = \leq_{k+1}$. Theorem of Nowakowski and Winkler says that the relation $\leq_k$ is total if and only if the graph is cop-win. On the other hand the number k (characteristic of the relational sequence) specifies the maximum length of the game the robber can provide playing against the perfect cop (but starting the chase in the worst possible vertex).

In the case of infinite graphs we may consider similar characterisation, but the sequence of the relations may be transfinite and hence it is indexed by the (not necessarily finite) ordinal numbers. Therefore, the characteristic of the sequence is some ordinal – we call it the CR-ordinal. Note that CR-ordinal no longer specifies the maximum game-time, it may be infinite while the length of the game is finite, but unbounded. We discuss the following question: which ordinals may be the CR-ordinals of infinite cop-wins graphs?

Joint work with Anthony Bonato and Gena Hahn.

Mariusz Grech

Semiregular automorphism groups of the graphical structures

We discuss which semiregular permutation groups are automorphism groups of graphs, digraphs, and supergraphs.

Eszter Gyimesi

A new combinatorial interpretation of r-Whitney numbers

T. A. Dowling introduced Whitney numbers of the first and second kind concerning the so-called Dowling lattices of finite groups. It turned out that they are generalizations of Stirling numbers. Later, I. Mező defined r-Whitney numbers as a common generalization of Whitney numbers and r-Stirling numbers.

Several authors gave different interpretations of these numbers. In our talk, we give a new combinatorial interpretation which correspond better with the combinatorial definition of Stirling and r-Stirling numbers. It allows us to derive new identities and properties of r-Whitney numbers.

Joint work with Gábor Nyul.

Carl Georg Heise

Nonempty intersection of longest paths in series-parallel graphs

In 1966 Gallai asked whether all longest paths in a connected graph have nonempty intersection. Even though this is not true in general, the answer to Gallai's question is positive for several well-known classes of graphs, e.g. outerplanar graphs, split graphs, and 2-trees. We present a proof that all series-parallel graphs (i.e. connected K_4-minor free graphs) have a vertex that is common to all of its longest paths.

Eliza Jackowska

The 3-colored Ramsey number of 3-uniform loose paths of length 3

In this talk we will sketch two proofs of the fact that the 3-colored Ramsey number of a 3-uniform loose path of length 3 is equal to 9.

The first proof is based on a case by case analysis of all colorings. The second proof relies on the determination of the Turán number $ex_3(9; P_3^3)$ and the corresponding extremal hypergraph.

This is one of the very few known results on 3-colored Ramsey numbers for hypergraphs. Other results of this kind were obtained by A. Gyárfás and G. Raeisi in [1].

References

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Katarzyna Jesse-Józefczyk

Graph decompositions and the expansion of secure sets

Consider a graph whose vertices play the role of members of the opposing groups. The edge between two vertices means that these vertices may defend or attack each other. At one time, any attacker may attack only one vertex. Similarly, any defender fights for itself or helps exactly one of its neighbours. If we have a set of defenders that can repell any attack, then we say that the set is secure. We examine the possibility of adding new members to the set so that it remains secure. We show that graph decomposition problems (e.g. matching cutset problem) are related to the problem of the expansion of secure sets.

Joint work with Elżbieta Sidorowicz.

Konstanty Junosza-Szaniawski

Fixing improper colorings of graphs

In this paper we consider some version of a recoloring problem. Let us have some non-proper r-coloring φ of a graph G. We investigate the problem of finding a proper r-coloring of G, which is "the most similar" to φ, i.e. the number k of vertices that have to be recolored is minimum possible. We prove that the problem is NP-complete for any $r \geq 3$, but is Fixed Parameter Tractable (FPT), when parametrized by the number of allowed transformations k. We provide an $\mathcal{O}^*(2^n)$ algorithm for the problem and a linear algorithm for graphs with bounded treewidth.

Finally, we investigate the *fixing number* of a graph G. It is the maximum possible distance (in the number of transformations) between some non-proper coloring of G and a proper one.

Mehdi Kadivar

Minimum flow problem, motivation and algorithms

Let G be a directed network defined by a set N of n nodes and a set A of m directed arcs. Capacities u_{ij} and l_{ij} are associated with each arc (i, j). The value u_{ij} denotes the maximum amount that can flow on the arc and l_{ij} denotes the minimum flow amount that must flow on the arc. We consider two special nodes, a source node s and a sink node t. The minimum flow problem is one of flow networks that computes the minimum flow between source and sink. There are several approaches to solve the problem but we will talk about decreasing path algorithms and preflow algorithms.

Vikram Kamat

Erdős-Ko-Rado theorems: stability analysis and a new generalizations

We first consider the following generalization of the seminal Erdős-Ko-Rado theorem, due to Frankl (1976). For k at least 2, call a family F of r-subsets of an n-element set k-wise intersecting if every subfamily of k sets from F has a nonempty intersection. Frankl proved a tight upper bound for k-wise intersecting families when $r \leq (k-1)n/k$. We prove a stability version of this theorem that provides structural information about large k-wise intersecting families, in particular for $r \geq n/2$. Next, we consider a higher order generalization of the EKR theorem. We prove a tight upper bound for intersecting families of matchings in the complete graph and also give a characterization of the extremal structures. The techniques used to prove these theorems are primarily inspired by Katona's method of cyclic permutations.

Hrant Khachatrian

On interval edge-colorings of complete multipartite graphs

An edge-coloring of a graph G with colors $1, \ldots, t$ is an *interval t-coloring* if all colors are used, and the colors of edges incident to each vertex of G are distinct and form an interval of integers [1]. A graph G is interval colorable if it has an interval t-coloring for some positive integer t. For an interval colorable graph G, the least and the greatest values of t for which G has an interval t-coloring are denoted by $w(G)$ and $W(G)$, respectively.

In this talk we present recent progress in the study of interval colorings of complete multipartite graphs. In particular, the colorability of several classes of complete tripartite graphs is determined, and some bounds for $w(G)$ and $W(G)$ are obtained.

Joint work with Andrzej Grzesik and Petros A. Petrosyan.

References

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Jakub Kozik

Multipass random greedy coloring of simple uniform hypergraphs

Let $\mathrm{Ds}(n)$ be the largest number such that every simple n-uniform hypergraph with maximum edge degree at most $\mathrm{Ds}(n)$ is two colorable (has Property B). We prove that $\mathrm{Ds}(n) = \Omega(n2^n)$. The method generalizes to b-simple hypergraphs (every pair of distinct edges intersects in at most b vertices) and hypergraphs of arithmetic progressions. This way we obtain an improved lower bound for van der Waerden numbers and get $W(n) = \Omega(2^n)$.

Joint work with Dmitry Shabanov.

Krzysztof Krzywdziński

Distributed algorithms and random graphs – MST problem

In many distributed systems, such as for example internet network, ad hoc networks or sensor networks, there are many entities, which operate in the system. Their processors are active at any moment and may perform some local computations. Moreover the entities have ability to communicate with each other to achieve some goals. The model of computation, which use the architecture of those systems is the distributed LOCAL model and algorithms in this model are called distributed.

Most of the known distributed algorithms either work on general graphs, i.e. study the worst cases, or concentrate on a particular family of graphs, such as for example: trees, bounded growth graphs, planar graphs or bounded degree graphs. In this talk we analyse the performance of distributed algorithms on average graphs. To this end we use an Erdős-Rényi random graph $G(n,p)$ with independent edges. A motivation for our research is fact that random graphs are commonly used tool to depict and analyse statistical behaviour of large networks. We will discuss the problem of constructing an approximation of a minimum spanning tree (MST) in $G(n,p)$ in LOCAL model. We will also mention other distributed algorithms for $G(n,p)$.

Daniela Kühn

On the typical structure of oriented graphs and digraphs with forbidden tournaments or cycles

There has been a huge body of work on the typical structure of H-free graphs, starting with a result of Erdős, Kleitman and Rothschild from 1976, who settled the case when H is a triangle. However, much less is known for oriented graphs or, more generally, for directed graphs.

Motivated by his work on the classification of countable homogeneous oriented graphs, Cherlin asked about the typical structure of oriented graphs (i) without a transitive triangle, or (ii) without an oriented triangle. We give an answer to these questions (which is not quite the predicted one).

Our approach is based on the recent 'hypergraph containers' method, developed independently by Saxton and Thomason as well as by Balogh, Morris and Samotij. Moreover, our results generalise to forbidden transitive tournaments and forbidden oriented cycles of any order, and also apply to digraphs. Along the way we prove several stability results for extremal digraph problems, which we believe are of independent interest. In my talk, I will survey these results and discuss several open problems.

Joint work with Deryk Osthus, Timothy Townsend and Yi Zhao.

Sławomir Kwasiborski

Fastest, average and quantile schedule

We consider problems concerning scheduling of trains on a single track. For every pair of trains there is minimal delay, which every train must wait before it enters the track after another train. Speeds of trains are also given. Hence for every scheduling – a sequence of trains – we may compute the time for all trains to travel along the track in the given order. We give the solution to three problems: the fastest schedule, average schedule and the problem of a quantile schedules. The last problem is a question about the smallest upper bound for time of a given fraction of all possible schedules. We show how these problems are related to travelling salesman problem. We prove NP-completeness of the fastest schedule problem and NP-harness of quantile of schedules problem and polinomiality of the average schedule. We also describe some algorithms for all three problems. In the solution of quantile problem we give an algorithm, based on ReverSearch method, generating with polynomial delay all eulerian multigraphs with the given degree sequence and a bound on the number of such multigraphs. A better bound is left as an open question.

Michał Lasoń

On the toric ideal of a matroid and related problems

Toric ideals is a certain class of ideals constructed using combinatorial data. When an ideal is defined only by combinatorial means, one expects to have a combinatorial description of its set of generators. An attempt to achieve this description often leads to surprisingly deep combinatorial questions.

White's conjecture is an example. It asserts that the toric ideal associated to a matroid is generated by quadratic binomials corresponding to symmetric exchanges. In the combinatorial language it means that if two multisets of bases of a matroid have equal union (as a multiset) then one can pass between them by a sequence of symmetric exchanges.

White's conjecture resisted numerous attempts since its formulation in 1980. We will discuss our recent results on White's conjecture and we will present some related combinatorial problems.

Joint work with Mateusz Michałek.

Nicolas Lichiardopol

Proof of a conjecture of Henning and Yeo on vertex-disjoint cycles

M.A. Henning and A. Yeo conjectured that a digraph of minimum out-degree at least 4, contains 2 vertex disjoint cycles of different lengths. In this paper we prove this conjecture. The main tool, is a new result (to our knowledge) asserting that in a digraph D of minimum out-degree at least 4, there exist 2 vertex-disjoint cycles C_1 and C_2, a path P_1 from a vertex x of C_1 to a vertex z not in $V(C_1) \cup V(C_2)$, and a path P_2 from a vertex y of C_2 to z, such that $V(P_1) \cap (V(C_1) \cup V(C_2)) = \{x\}$, $V(P_2) \cap (V(C_1) \cup V(C_2)) = \{y\}$, and $V(P_1) \cap V(P_2) = \{z\}$ (we say then that D has property (P)). In the last section, a conjecture will be proposed.

Ivica Martinjak

Polarized partitions and partitions with d-distant parts

A partition with d-distant parts is the sequence $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)$ with the constraint $\lambda_i - \lambda_{i+1} \geq d$. These partitions are generalization of partitions of Rogers-Ramanujan type. Partitions with 1-distant parts and with the property that the minimal even part is greater than double number of odd parts we call *polarized*. It is known that the number of polarized partitions $\lambda \vdash n$ is equal to the number of partitions with 2-distant parts, as a specialization of a more general bijection [3]. We prove that the number of partitions $\lambda \vdash n$ with d-distant parts is equal to the number of polarized partitions $\mu_i \vdash n-(d-2)\binom{i}{2}$, with length equal to i, $i \geq 1$ [4]. As a consequence of this result, the number of partitions with d-distant parts is represented as the number of partitions of Rogers-Ramanujan type.

Joint work with Dragutin Svrtan.

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Aleksander Mądry

From graphs to matrices, and back: bridging the combinatorial and the continuous

Graphs are ubiquitous in all modern sciences. This motivates a need for algorithmic tools that are capable of analyzing and computing on graphs in an efficient manner. A need that is even more pressing now that the graphs we are dealing with tend to be massive, rendering traditional methods no longer adequate.

In this talk, I will discuss a recent progress on the maximum flow problem and use it as an illustration of a broader emerging theme in graph algorithms that employs optimization methods as an algorithmic bridge between our combinatorial and spectral understanding of graphs.

I will also briefly outline how this line of work brings a new perspective on some of the core continuous optimization primitives – most notably, interior-point methods.

Piotr Micek

On the dimension of posets with cover graphs of treewidth 2

In 1977, Trotter and Moore proved that a poset has dimension at most 3 whenever its cover graph is a forest, or equivalently, has treewidth at most 1. On the other hand, a well-known construction of Kelly shows that there are posets of arbitrarily large dimension whose cover graphs have treewidth 3. In this paper we focus on the boundary case of treewidth 2. It was recently shown that the dimension is bounded if the cover graph is outerplanar (Felsner, Trotter, and Wiechert) or if it has pathwidth 2 (Biró, Keller, and Young). This can be interpreted as evidence that the dimension should be bounded more generally when the cover graph has treewidth 2. We show that it is indeed the case: Every such poset has dimension at most 2554.

Joint work with Gwenaël Joret, William T. Trotter, Ruidong Wang and Veit Wiechert.

Mirjana Mikalački

How fast can Maker win in biased games on $E(K_n)$?

We study Maker–Breaker games played on the edge set of the complete graph on n vertices, K_n, for sufficiently large integer n. In the $(a : b)$ Maker–Breaker game, two players, *Maker* and *Breaker*, take turns in claiming previously unclaimed edges of K_n, where Maker claims a edges, and Breaker claims b edges per turn. Maker wins the game, if by the end of the game he occupies all the elements of one winning set. Otherwise, Breaker wins. We look at the $(1 : b)$ *Perfect matching game* and $(1 : b)$ *Hamiltonicity game*, where the winning sets are the edge sets of all perfect matchings on $E(K_n)$, and the edge sets of all Hamilton cycles on K_n, respectively. For these two games, we are interested in determining the least number of moves that Maker needs to make in order to win and, on the other hand, for how long can Breaker delay Maker's win.

Joint a work with Asaf Ferber, Dan Hefetz and Miloš Stojaković.

Piotr Miska

Arithmetic properties of sequence of derangements and its generalizations

The sequence of derangements is given by a formula

$$D_0 = 1, \quad D_n = nD_{n-1} + (-1)^n \text{ for } n > 0.$$

It is a classical object appearing in combinatorics and number theory.

For each positive integer n we have $n-1 \mid D_n$. In particular, p-adic valuation of D_n is estimated from below by p-adic valuation of $n-1$ for each prime number p. We prove that there are infinitely many prime numbers p such that $v_p(D_n) < v_p(n-1)$ for some positive integer n. Moreover, we give the description of p-adic valuation of $\frac{D_n}{n-1}$ for a given prime number p.

We show that for each positive integer k there exists polynomial $f_k \in \mathbb{Z}[X]$ such that $D_{n+k} = (n+1)\dots(n+k)D_n + (-1)^n f_k(n)$ for all positive integers n. Furthermore we prove that f_k has exactly $k-1$ distinct real roots, among which there is exactly one rational root $1-k$.

Furthermore we generalize the properties above for sequences given by a formula

$$a_0 = h_1(0), \quad a_n = f(n)a_{n-1} + h_1(n)h_2(n)^n \text{ for } n > 1,$$

where $f, h_1, h_2 \in \mathbb{Z}[X]$. If time permits we present further extensions of our results.

Khandoker Mohammed Mominul Haque

Irregular total labelling of Möbius ladder M_n

The total edge irregularity strength tes(G) and total vertex irregularity strength tvs(G) are invariants analogous to irregular strength $s(G)$ of a graph G for total labellings. Bača et al. [Discrete Mathematics 307, 2007, 1378–1388] determined the bounds and precise values for some families of graphs concerning these parameters. In this paper, we show the exact values of the total edge irregularity strength is $\mathrm{tes}(M_n) = n+1$ and total vertex irregularity strength is $tvs(M_n) = \lceil n/2 \rceil + 1$ for the Möbius ladder M_n.

Gábor Nyul

Generalization of combinatorial numbers for graphs

B. Duncan and R. Peele defined Stirling numbers of the second kind and Bell numbers for graphs. In our talk, we give an extensive study of them, furthermore, we introduce and investigate Fubini numbers for graphs. For special graphs, they give back the ordinary, nonconsecutive and r-generalized variants of these numbers.

Joint work with Zsófia Kereskényi-Balogh.

Deryk Osthus

Decompositions of graphs into small subgraphs

In this talk, I will discuss recent progress on the problem of decomposing a large graph into small or sparse subgraphs.

A fundamental theorem of Wilson states that, for every graph F, every sufficiently large F-divisible clique has an F-decomposition. Here a graph G is F-divisible if $e(F)$ divides $e(G)$ and the greatest common divisor of the degrees of F divides the greatest common divisor of the degrees of G, and G has an F-decomposition if the edges of G can be covered by edge-disjoint copies of F. We extend Wilson's theorem to graphs which are allowed to be far from complete (joint work with B. Barber, D. Kühn, A. Lo).

I will also discuss decompositions of dense graphs and hypergraphs into Hamilton cycles and perfect matchings, including the proof of the 1-factorization conjecture and the Hamilton decomposition conjecture.

Joint work with Béla Csaba, Daniela Kühn, Allan Lo and Andrew Treglown.

Monika Pilśniak

Distinguishing graphs by edge colourings

We say that a (not necessatily proper) colouring c of edges of a graph G *breaks an automorphism* φ if there exists an edge $xy \in E(G)$ such that $c(\varphi(x)\varphi(y)) \neq c(xy)$. A *distinguishing index* $D'(G)$ of a graph G is the least number d such that G admits an edge colouring with d colours that breaks all nontrivial automorphisms. This is an analogue to the notion of the *distinguishing number* $D(G)$ of a graph G, which was introduced by Albertson and Collins for vertex colourings.

We investigate upper bounds for the distinguishing index in general, and for some classes of finite and infinite graphs, e.g. trees, traceable or 3-connected planar graphs.

Joint work with Rafał Kalinowski.

Alexey Pokrovskiy

Linkage structures in tournaments

A (possibly directed) graph is k-linked if for any two disjoint sets of vertices $\{x_1, \ldots, x_k\}$ and $\{y_1, \ldots, y_k\}$ there are vertex disjoint paths $P_1, \ldots, P_k$ such that P_i goes from x_i to y_i. A theorem of Bollobás and Thomason says that every $22k$-connected (undirected) graph is k-linked. It is desirable to obtain analogues for directed graphs as well. Although Thomassen showed that the Bollobás-Thomason Theorem does not hold for general directed graphs, he proved an analogue of the theorem for tournaments – there is a function $f(k)$ such that every strongly $f(k)$-connected tournament is k-linked. The bound on $f(k)$ was reduced to $O(k \log k)$ by Kühn, Lapinskas, Osthus and Patel, who also conjectured that a linear bound should hold. We will discuss a proof of this conjecture.

Michał Przykucki

The time of graph bootstrap percolation

Graph bootstrap percolation, introduced by Bollobás in 1968, is a cellular automaton defined as follows. Given a "small" graph H and a "large" graph $G = G_0 \subseteq K_n$, in consecutive steps we obtain G_{t+1} from G_t by adding to it all new edges e such that $G_t \cup e$ contains a new copy of H. We say that G percolates if for some $t \geq 0$ we have $G_t = K_n$.

For $H = K_r$, the question about the smallest size of percolating graphs was independently answered by Alon, Frankl and Kalai in the 1980's. Recently, Balogh, Bollobás and Morris considered graph bootstrap percolation for $G = G(n, p)$ and studied the critical probability $p_c(n, K_r)$ for which the graph percolates with high probability. In this paper, using the same setup, we determine up to a logarithmic factor the critical probability for percolation by time t for all $1 \leq t \leq \log\log\log n$.

Joint work with Karen Gunderson and Sebastian Koch.

Krzysztof Pszczoła

On some properties of the k-transitive closure of a directed path

In our talk we will define the k-transitive closure of the oriented graph and discuss some properties of the k-transitive closure of a directed path. In particular we calculate the density of such graphs and observe that the 3-transitive closure of a directed path of odd number of vertices is irregular.

Philipp Pushnyakov

Around Turán's theorem

Let G be an undirected graph with independance number $\alpha = \alpha(G)$. We want to estimate the number of edges in an arbitrary set of vertices A. For example, if $l = |A| \leq \alpha$ then A may contain no edge, but if $l = |A| > \alpha$ then A contains at least one edge. In 1941 Turán showed that if $l = |A| > \alpha$ then $r(W) \geq \frac{l^2}{2\alpha} - \frac{l}{2}$, where $r(W)$ is the number of edges between vertices from A. In our work we improve this result for special distance graphs.

Stanisław Radziszowski

Ramsey arrowing of triangles

In 1967, Erdős and Hajnal posed a challenge to construct graphs without small cliques whose every edge-coloring contains monochromatic triangles. The most basic related question is: Does there exist a K_4-free graph that is not a union of two triangle-free graphs? The answer involves solving a special instance of the classical Ramsey arrowing predicate for triangles. In 1970, Folkman proved that such graphs exist, but far from providing their effective construction. During the following 44 years of slow but steady progress we learned a few new insights about this Ramsey triangle arrowing. In this talk we will overview them and discuss some related problems.

Ghaffar Raeisi

Ramsey number of disjoint copies of linear hypergraphs

A k-uniform hypergraph H is a pair (V, E) where V is a set of vertices and E is a set of k-subsets of V (the edges of H). A hypergraph H is linear if the intersection of every two edges of H has at most one element. For given k-uniform hypergraphs G and H, the Ramsey number $R(G, H)$ is the smallest positive integer N such that in every red-blue coloring of the edges of the complete k-uniform hypergraph on N vertices there is a monochromatic copy of G in color red or a monochromatic copy of H in color blue. In this talk, for sufficiently large n and m, we determine the Ramsey number $R(mG, nH)$ where G and H are linear k-uniform hypergraphs. As an application, $R(mG, nH)$ is determined where m or n is large and G and H are either k-uniform loose paths, loose cycles or stars.

Anitha Rajkumar

The pseudograph (r, s, a, t)-threshold number

For $d \geqslant 1$, $s \geqslant 0$, a $(d, d+s)$-graph is a graph whose degrees all lie in the interval $\{d, d+1, \ldots, d+s\}$. For $r \geqslant 1$, $a \geqslant 0$, an $(r, r+a)$-factor of a graph G is a spanning $(r, r+a)$-subgraph of G. An $(r, r+a)$-factorization of a graph G is a decomposition of G into edge-disjoint $(r, r+a)$-factors. A pseudograph is a graph which may have multiple edges and may have multiple loops. A loop counts two towards the degree of the vertex it is on. A multigraph here has no loops.

For $t \geqslant 1$ let $\pi(r, s, a, t)$ be the least integer such that, if $d \geqslant \pi(r, s, a, t)$, then every $(d, d+s)$-pseudograph G has an $(r, r+a)$-factorization into x $(r, r+a)$-factors for at least t different values of x. We call $\pi(r, s, a, t)$ the pseudograph (r, s, a, t)-threshold number. Let $\mu(r, s, a, t)$ be the analogous integer for multigraphs. We call $\mu(r, s, a, t)$ the multigraph (r, s, a, t)-threshold number. A simple graph has at most one edge between any two vertices and has no loops. We call $\sigma(r, s, a, t)$ be the analogous integer for simple graphs. We call $\sigma(r, s, a, t)$ the simple graph (r, s, a, t)-threshold number.

In this talk we will give the precise value of the pseudograph $\pi(r, s, a, t)$-threshold numbers for each value of r, s, a and t. Also we use this to give good bounds for the analogous simple graph and multigraph threshold numbers $\sigma(r, s, a, t)$ and $\mu(r, s, a, t)$.

Katarzyna Rybarczyk

The chromatic index of random hypergraphs

Let $H_d(n, M)$ be a random hypergraph on n vertices and with M edges of size d. We will show some results concerning the chromatic index of $H_d(n, M)$. We will be interested in the case $d \to \infty$ and we will study the performance of the greedy algorithm constructing a colouring. Motivation for our research is to determine the chromatic number of random intersection graph $G(M, n, d)$. We will recall the results concerning the stability number of $G(M, n, d)$ and discuss the relation between them and the chromatic index of $H_d(n, M)$.

Paweł Rzążewski

(Pseudo)achromatic coloring of fragmentable hypergraphs

A complete c-coloring of a k-uniform hypergraph $H = (V, \mathcal{E})$ is a mapping $f \colon V \to [c]$ such that every k-subset of colors appears on some hyperedge. The maximum c for which there exists a complete c-coloring of H is called the *pseudoachromatic number* of H and denoted by $\psi'(H)$. An *achromatic number* of H (denoted by $\psi(H)$) is the maximum c, for which H admits a complete c-coloring in which no two vertices belonging to the same hyperedge have the same color. By calculating the number of possible color patterns, we obtain that $\psi(H) \leq \sqrt[k]{k!m}$, where m is the number of hyperedges of H.

In this talk we consider a pseudoachromatic number and an achromatic number of *fragmentable* hypergraphs. Intuitively, a class Γ of hypergraphs is fragmentable if every hypergraph from Γ can be split into disjoint subhypergraphs of constant order by removing a small fraction of vertices. We provide an asymptotically tight bound on the achromatic number of uniform, regular, fragmentable hypergraphs. Formally, we prove the following theorem.

Theorem 1 *Fix $\epsilon > 0$ and integers k and Δ. Let Γ be a fragmentable class of hypergraphs. Every k-uniform Δ-regular hypergraph $H \in \Gamma$ with m hyperedges (for sufficiently large m) admits a complete coloring with at least $(1-\epsilon)\sqrt[k]{k!m}$ colors.*

Our approach is basically non-constructive. The key ingredient is the result on covering uniform regular hypergraphs, proven by the so-called Rödl nibble method.

Tina Janne Schmidt

On the relationship of large minimum bisection and tree width in planar graphs

Minimum Bisection denotes the NP-hard problem to partition the vertex set of a graph into two classes of equal size while minimizing the number of edges between these sets. Here, we focus on bounded degree planar graphs. It is known that such graphs admit a bisection of width $O(\sqrt{n})$, where n denotes the number of vertices. Furthermore, if such a graph has large minimum bisection width, that is, if it admits no bisection of width less than $c\sqrt{n}$ for some fixed $c > 0$, then it has large tree width and hence it contains a large grid as minor. The converse is not true. Here, we present sufficient conditions such that large tree width implies a large minimum bisection width.

Joint work with Cristina G. Fernandes and Anusch Taraz.

Joanna Sokół

Fractional and j-fold colouring of the plane

We present results referring to the Hadwiger-Nelson problem which asks for the minimum number of colours needed to colour the plane with no two points at distance 1 having the same colour. Exoo considers a more general problem concerning graphs $G_{[a,b]}$ with $\mathbb{R}^2$ as the vertex set and two vertices adjacent if their distance is in the set $[a,b]$. Exoo proved that $\chi(G_{[a,b]}) \geq 5$ for $b > 1.3114a$ and conjectured $\chi(G_{[a,b]}) \geq 7$ for $b > a$. We partially answer this conjecture by proving that $\chi(G_{[a,b]}) \geq 5$ for $b > a$.

A *j-fold colouring* of a graph $G = (V, E)$ is an assignment of j-element sets of colours to the vertices of G, in such a way that the sets assigned to any two adjacent vertices are disjoint. The smallest number of colours required for j-fold colouring of a graph G is called *j-fold chromatic number* $\chi_j(G)$. The *fractional chromatic number* is defined to be: $\chi_f(G) = \inf \chi_j(G)/j$.

We present some bounds for the j-fold chromatic number of $G_{[a,b]}$ for small j, in particular for $G_{[1,1]}$ and $G_{[1,2]}$. The j-fold colouring for small j has strong practical motivation especially in scheduling theory, while graph $G_{[1,2]}$ is often used to model hidden conflicts in radio networks. Moreover, we generalize a method by Hochberg and O'Donnell (who proved that $G_{[1,1]} \leq 4.36$) for fractional colouring of graph $G_{[a,b]}$, obtaining $\chi_f(G_{[1,2]}) \leq 9.9$.

Katherine Staden

Embedding squares of Hamilton cycles via degree sequence conditions

Many famous results in extremal graph theory give minimum degree conditions that force some substructure. For example, Dirac's classical theorem characterises the minimum degree that ensures a Hamilton cycle in a graph. However, sometimes it is possible to obtain stronger results via *degree sequence* conditions. For example, Pósa gave a significant strengthening of Dirac's theorem: if $d_1 \leq \ldots \leq d_n$ is the degree sequence of G and $d_i > i$ for all $1 \leq i < n/2$, then G contains a Hamilton cycle. A famous conjecture of Pósa gave a minimum degree condition that ensures a graph contains the square of a Hamilton cycle. This was proved for large n by Komlós, Sárközy and Szemerédi. In this talk we consider a degree sequence analogue of this theorem.

Michał Stronkowski

Finite axiomatization for quasivarieties of digraphs

Let $\mathbf{D}$ be a finite digraph and $\mathsf{Q}(\mathbf{D})$ be the quasivariety generated by $\mathbf{D}$. Recall that $\mathsf{Q}(\mathbf{D})$ is the class of digraphs isomorphic to (induced) subdigraphs of direct powers of $\mathbf{D}$. We study the following problem.

Problem 1 *Is it decidable whether for a given digraph* $\mathbf{D}$ *the quasivariety* $\mathsf{Q}(\mathbf{D})$ *may be defined by a sentence in first order logic?*

This is a variation of Tarski's finite basis problem. It asks whether the existence of a finite axiomatization for a variety generated by a finite algebra is decidable. Tarski's finite basis problem was solved negatively by Ralph McKenzie. However its quasivarietal variant, for algebras or relational structures, is open.

We will survey what is known about the problem. Then we will present some new results for oriented paths.

Joint work with Marcel Jackson and Tomasz Kowalski.

Benny Sudakov

Grid Ramsey problem and related questions

The Hales-Jewett theorem is one of the pillars of Ramsey theory, from which many other results follow. A celebrated result of Shelah says that Hales-Jewett numbers are primitive recursive. A key tool used in his proof, known as the cube lemma, has become famous in its own right. In its simplest form, it says that if we color the edges of the Cartesian product $K_n \times K_n$ in r colors then, for n sufficiently large, there is a rectangle with both pairs of opposite edges receiving the same color.

Hoping to improve Shelah's result, Graham, Rothschild and Spencer asked more than 20 years ago whether the cube lemma holds with n which is polynomial in r. We show that this is not possible by providing a superpolynomial lower bound in r. We also discuss a number of related questions, among them a solution of a problem of Erdős and Gyárfás on generalized Ramsey numbers.

Joint work with David Conlon, Jacob Fox and Choongbum Lee.

Małgorzata Sulkowska

Percolation and best choice problem for powers of paths

The vertices of a kth power of a directed path of length n are exposed one by one to a selector in some random order. At any time the selector can see the graph induced by the vertices that have already come and gets some extra information about the edges that have already emerged. The selector's aim is to choose on-line the maximal vertex (i.e., the vertex with no outgoing edges). We give the exact asymptotic behaviour of the probability of success p_n of the optimal algorithm: $\lim_{n\to\infty} p_n n^{1/(k+1)} = \Gamma(1 + 1/(k+1))$. In order to prove this result we analyze a site percolation process on a sequence of kth powers of a directed path of length n.

Magdalena Szymkowiak

A randomized k tracks variant of the Gate Matrix Layout problem

In this talk the k-Gate Matrix Layout (k-GML) problem, as a simple layout style for very large scale integration (VLSI) design, will be presented. The problem asks if k tracks are enough to lay out a given circuit. It may be formulated in terms of random Boolean matrices, in terms of random bipartite graphs, as well as, in terms of random intersection graphs.

Our goal is to find thresholds for the appearance of matrices which are 'yes' instances for a randomized k-GML problem.

Amelia Taylor

Arbitrary orientations of Hamilton cycles in digraphs

Let n be sufficiently large and suppose that G is a digraph on n vertices where every vertex has in- and outdegree at least $n/2$. We show that G contains every orientation of a Hamilton cycle except, possibly, the antidirected one. The antidirected case was settled by DeBiasio and Molla, where the threshold is $n/2 + 1$. Our result is best possible and improves on an approximate result by Häggkvist and Thomason.

Joint work with Louis DeBiasio, Daniela Kühn, Theodore Molla and Deryk Osthus.

Dirk Oliver Theis

Nondeterministic communication complexity of random 01 matrices

A construction of Lovász and Saks gives a graph $G(M)$ for each 01 matrix M in such a way that the nondeterministic communication complexity of the matrix equals the binary logarithm of the chromatic number of the graph. The graph $G(M)$ is called the Lovász-Saks graph of the matrix M.

This talk is about the chromatic number of Lovász-Saks graphs of n by n random 01 matrices with independent entries. We present results and several open problems.

Timothy Townsend

Proof of a tournament partition conjecture and an application to 1-factors with prescribed cycle lengths

In 1982 Thomassen asked whether there exists an integer $f(k,t)$ such that every strongly $f(k,t)$-connected tournament T admits a partition of its vertex set into t vertex classes $V_1, \ldots, V_t$ such that for all i the subtournament $T[V_i]$ induced on T by V_i is strongly k-connected. Our main result implies an affirmative answer to this question. In particular we show that $f(k,t) = O(k^7t^4)$ suffices. As another application of our main result we give an affirmative answer to a question of Song as to whether, for any integer t, there exists an integer $h(t)$ such that every strongly $h(t)$-connected tournament has a 1-factor consisting of t vertex-disjoint cycles of prescribed lengths. We show that $h(t) = O(t^5)$ suffices.

Joint work with Daniela Kühn and Deryk Osthus.

Michał Tuczyński

Counting independent sets via Divide, Measure and Conquer method

We give an algorithm for counting the number of all independent sets in a given graph which works in time $O^*(1.1393^n)$ for subcubic graphs and in time $O^*(1.2369^n)$ for general graphs, where n is the number of vertices in the instance graph, and polynomial space. The result comes from combining two well known methods Divide and Conquer and Measure and Conquer. So we introduce this new concept of Divide, Measure and Conquer method and expect it will find applications in other problems.

Algorithm of Björklund, Husfeldt and Koivisto for graph colouring with our algorithm used as a subroutine has complexity $O^*(2.2369^n)$ and is currently the fastest graph colouring algorithm in polynomial space.

Zsolt Tuza

Choice identification in graphs

In a graph G, choice identification means to select a set S of vertices and, for each vertex v, to specify a subset $f(v)$ of S which is also a subset of the closed neighborhood of v, with the further condition that the specified subsets are distinct for all vertices. The goal is to determine or estimate the minimum cardinality of a set S which can be taken in a choice identification in G. This graph invariant was recently introduced by T.-P. Chang and L.-D. Tong in [1]. We present results on algorithmic complexity and on relations to other graph invariants.

Joint work with Cristina Bazgan and Csilla Bujtás.

References

[1] T.-P. Chang, L.-D. Tong, *Choice identification of a graph*, Discrete Applied Math. 167, 2014, 61–71.

Torsten Ueckerdt

On-line coloring between two lines

We consider intersection graphs of connected objects spanned between two parallel lines. Restricting the possible shapes of objects to be convex sets, we obtain a generalization of permutation, simple-triangle, triangle and trapezoid graphs. We propose an on-line coloring algorithm for this general case using $O(\chi^3)$ colors for graphs with chromatic number χ. The best known lower bound construction due to Szemerédi gives $\Omega(\chi^2)$ even for permutation graphs. We also consider graphs induced by curves between the two lines with pairwise bounded number of intersections.

Bartosz Walczak

Coloring geometric intersection graphs and related problems

Colorings of graphs represented by geometric objects have been studied since the 1960s for purely aesthetic as well as practical reasons. One of them is that geometric representations provide natural examples of classes of graphs that are near-perfect in the sense that their chromatic number is bounded by a function of their clique number. For many years, it was conjectured that all classes of geometric intersection graphs in the plane have this property, but this turned out to be false. I will discuss recent progress in the study of this and other problems on geometric intersection graph colorings, explain the connection with on-line graph coloring algorithms that was one of the driving forces of this progress, and review some related problems of not necessarily geometric nature.

Krzysztof Węsek

Avoiding repetitions on the plane

We present results in the intersection of two areas of combinatorics, both full of interesting results, challenging open problems and various applications: colorings of geometrical structures (inspired by the Hadwiger-Nelson problem of proper coloring unit distance graph of the plane) and combinatorics on words (inspired by the classic theorem of Thue that there exists an arbitrarily long sequence avoiding 2-repetitions on only 3 symbols).

An *r-repetition* is a sequence $a_1, a_2, \dots, a_{rn}$ such that $a_i = a_{i+jn}$ for all $i = 1, \dots, n$ and $j = 1, \dots, r-1$. We solved a problem of Grytczuk by showing that a countable number of colors is insufficient to avoid repetitions on every simple path of the unit distance graph of the plane.

This infiniteness leads to relaxing the condition by examining only 'some' paths of the unit distance graph of the plane. We take into account only *line paths* - sequences of collinear points where consecutive points are in distance one. The coloring of the plane is *line r-nonrepetitive* if colors of no line path determines an r-repetition. It was known that 36 colors suffice for line 2-coloring of the plane – we have improved this bound to 18 by using a special rectangle tiling of the plane combined with a help of the classic Thue sequence. Additional result of this work concerns palindromes: a *palindrome* is a sequence that is the same if we reverse the order of elements. We proved that there exists a 32-coloring of the plane that is line 2-nonrepetitive and *line palindrome-free* (colors of no line path determines a nontrivial palindrome).

Moreover, we also studied line r-repetitions on the plane and, in general, on $\mathbb{R}^n$ for any positive n. We managed to prove that for sufficiently large $r(n)$ there exists a line $r(n)$-nonrepetitive coloring of the plane using only 2 colors – in case of $\mathbb{R}^2$ it is enough to take $r = 47$.

Kristiana Wijaya

On Ramsey (mK_2, H)-minimal graphs

Let F, G and H be non-empty graphs. The notation $F \rightarrow (G, H)$ means that if all edges of F are arbitrarily colored by red or blue then either the red subgraph of F contains a graph G or the blue subgraph of F contains a graph H. A graph F satisfying $F \rightarrow (G, H)$ and $(F - e) \nrightarrow (G, H)$ for every $e \in E(F)$ is called a *Ramsey* (G, H)*-minimal graph*. The set of all Ramsey (G, H)-minimal graphs is denoted by $\mathfrak{R}(G, H)$. In this paper, we derive the necessary and sufficient condition of graphs belonging to $\mathfrak{R}(mK_2, H)$ for an arbitrarily connected graph H. Furthermore, we give a method to generate a connected graph in $\mathfrak{R}((m + 1)K_2, P_3)$ by operation subdivision non pendant edge of a connected graph in $\mathfrak{R}(mK_2, P_3)$.

Rafał Witkowski

De Bruijn sequence in 2D

A k-ary de Bruijn sequence $B(k,n)$ of order n, is a sequence over a given alphabet A with size k for which every subsequence of length n in A appears as a sequence of consecutive characters exactly once. De Bruijn matrix is an array of symbols from an alphabet A of size k that contains every $m \times n$ matrix exactly once. Since we know a lot about de Bruijn sequences, there is no much work made about de Bruijn matrices. During the talk we will say about some generalization of de Bruijn sequences into 2 dimensions.